Motion in a plane

1. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $5\pi/36$ rad would strike a given target. When the shell was fired it was found that a wall just prevented the projectile from completing its trajectory. At what angle of elevation should the gun be fired to hit the target?

   a) $6\pi/36$ rad b) $11\pi/36$ rad c) $7\pi/36$ rad d) $13\pi/36$ rad

2. A cannon is aimed at an angle θ above the horizontal and a shell is fired with a muzzle velocity v_o towards a vertical wall that is at a distance D from the cannon. The height from the bottom of the wall at which the shell strikes it is

   a) $D\tan^2(\theta) - \dfrac{gD^2}{v_o^2\cos^2(\theta)}$ b) $D\tan(\theta) + \dfrac{gD^2}{2v_o^2\cos^2(\theta)}$

   c) $D\tan(\theta) - \dfrac{gD^2}{2v_o^2\cos^2(\theta)}$ d) $D\tan(\theta) + \dfrac{gD^2}{2v_o^2\cos(\theta)}$

3. A particle is projected from a point P with a velocity v at an angle θ with horizontal. At a certain point Q it moves at right angle to its initial direction. Then velocity of particle at Q and time of flight from P to Q are

   a) $v\cot(\theta)$ and $v\,\text{cosec}(\theta)/g$ b) $v\cos(\theta)$ and $v\sec(\theta)/g$

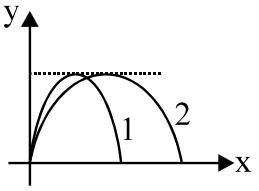
   c) $v\,\text{cosec}(\theta)$ and $v\sin(\theta)/g$ d) zero and $v\,\text{cosec}(\theta)/g$

4. A body is projected with velocity u at an angle of projection θ with the horizontal. The body makes 30° w.r.t. the horizontal at t = 2 s. One second later it reaches the maximum height. Initial velocity and angle of projection are

   a) $20\sqrt{3}$ ms^{-1} and 45° b) $30\sqrt{3}$ ms^{-1} and 60°

   c) $10\sqrt{3}$ ms^{-1} and 30° d) $20\sqrt{3}$ ms^{-1} and 60°

5. Trajectories of two projectiles are shown in figure. Let T_1 and T_2 be the time period and u_1 and u_2 their speeds of projection. Choose the correct options from the following



   a) $T_1 = T_2$ & $u_1 < u_2$ b) $T_1 > T_2$ & $u_1 = u_2$

   c) $T_1 > T_2$ & $u_1 < u_2$ d) $T_1 < T_2$ & $u_1 > u_2$

6. A ball is thrown from ground level so as to just clear a wall 4 meters high at a distance of 4 m and falls at a distance of 14 m from the wall. Initial velocity of the ball and its angle of projection are respectively

   a) $\sqrt{3}g/2$ and 45° b) $2g\sqrt{3}$ and 60°

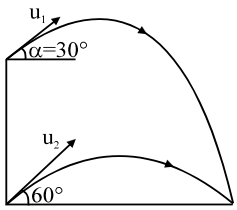
   c) $2g/\sqrt{3}$ and 15° d) $g\sqrt{3}$ and 30°

7. A projectile is launched at an angle θ from a cliff of height H above the sea level. If it falls into the sea at a distance D from the base of the cliff then the maximum height attained by it, above the sea level, is

a) $H + \frac{D^2\cot^2(\theta)}{\sqrt{2}(D+H\tan\theta)}$ b) $H + \frac{D^2\tan^2(\theta)}{\sqrt{3}(H+D\sin\theta)}$

b) $D + \frac{H^2\sin^2(\theta)}{4(H+D\tan\theta)}$ d) $H + \frac{D^2\tan^2(\theta)}{4(H+D\tan\theta)}$

8. Particles are fired simultaneously from the top and bottom of a vertical tower with the elevation $\theta_1 = 30°$, $\theta_2 = 60°$ respectively and particles strike simultaneously at the same point when the reach the ground. If the distance where they hit the ground is $30\sqrt{3}$ m from the foot of the tower then (more than one options may be correct)



a) Time of flight is $3\sqrt{2}$
b) Height of the tower is 60 m
c) u_1 is $10\sqrt{6}$
d) u_2 is $10\sqrt{2}$

9. A ball is projected horizontally with a speed v from the top of a plane inclined at an angle 45° with the horizontal. How far from the point of projection with the ball strike the plane?

a) $\frac{v^2}{g}$ b) $\sqrt{2}\frac{v^2}{g}$ c) $\frac{2v^2}{g}$ d) $\frac{2\sqrt{2}v^2}{g}$

10. A particle projected with velocity u strikes an inclined plane normally at point P. If the angle of inclination of the plane w.r.t. the ground is β then (more than one options may be correct)

a) Height of the point P w.r.t. the ground is $\frac{2u^2}{g}\left(\frac{\sin^2\beta}{1+3\sin^2\beta}\right)$

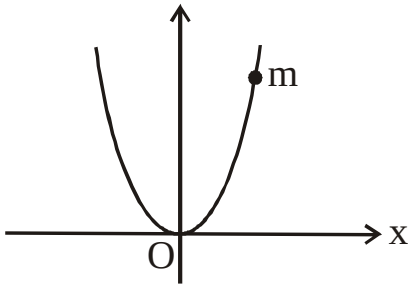
b) Time of flight of the particle is $\frac{2u}{g\sqrt{1+3\sin^2\beta}}$

c) Time of descent is $\frac{u}{g\sqrt{1+3\sin^2\beta}}$

d) The maximum height reached by the particle w.r.t. the ground is $\frac{u^2}{g}\left(\frac{\sin^2\beta}{1+3\sin^2\beta}\right)$

11. A bead of mass m is located on a parabolic smooth fixed wire with its axis vertical and vertex at the origin as shown in figure. Shape of the wire is given by the relation $x^2 = 4Ay$. The bead is

released from rest at point $y = 4A$ on the wire frame. Tangential acceleration of the bead when it reaches the position given by $y = A$ is



a) $g/\sqrt{2}$ b) $3g/\sqrt{2}$ c) $g\sqrt{2}/3$ d) zero

12. An object is moving in the xy plane with the position as a function of time given by the relation $\boldsymbol{r} = x(t)\hat{i} + y(t)\hat{j}$ .If point O is at $\boldsymbol{r}$ = 0 then the distance of object from O is definitely decreasing when

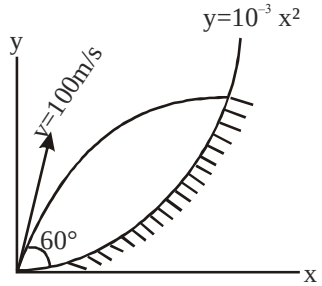
a) $v_x > 0$ and $v_y > 0$ b) $v_x < 0$ and $v_y < 0$ c) $xv_x + yv_y < 0$ d) $xv_x + yv_y > 0$

13. Two stones are projected from origin (towards the same side of the origin) with the same speed of 20 ms^{-1} One of the stones is projected an angle θ w.r.t the horizontal and the other at angle θ w.r.t. the vertical. Find the distance between the stones 1 second after projection. (Take $\cos(\theta)$ as 4/5 and $\sin(\theta)$ as 3/5).

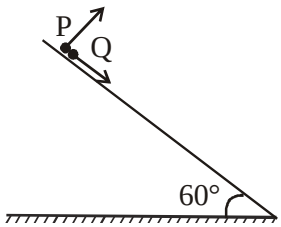
14. A particle is projected up an inclined plane such that its component of velocity along the incline is 10 ms^{-1}. If its time of flight is 2 seconds and maximum height above the incline is 5 m then find the initial velocity of projection.

15. A ball is thrown upward with initial velocity of 15.0 ms^{-1} at an angle of 30° w.r.t. the horizontal. If the point of projection is at the top of an inclined plane which slopes downward at an angle of 20° then the time after which the ball strikes the slope is _____

16. A projectile is fired at an angle of 60° with muzzle velocity 100 ms^{-1} towards the curved surface of a hill as shown in the figure. At what elevation (y) does it strike the hill? A line in the vertical plane of the curved surface is given by the relation $y = 10^{-3} x^2$ (g = 10 ms^{-2})



17. A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide on the inclined plane after t = 4 second. What is the speed of projection of P?



18. A particle is projected with a velocity u at an angle θ with the horizontal. Prove that the radius of curvature of its path, at the point where velocity makes an angle $\theta/2$ with the horizontal is given the relation $\frac{u^2 \cos^2\theta}{g \cos^3(\theta/2)}$

19. If R is the horizontal range and h is the maximum height reached by the projectile for an angle of projection θ, then prove that the maximum possible range is given by the relation $R_{max} = \frac{R^2}{8h} + 2h$

20. If T is the total time of flight, H the maximum height and R is the horizontal range of a projectile. Prove that

    (a) y coordinate of the projectile is given as a function of t and T by the relation $y = 4H\left(\frac{t}{T}\right)\left(1-\frac{t}{T}\right)$

    (b) y coordinate of the projectile is given as a function of x and R by the relation $x = 4H\left(\frac{x}{R}\right)\left(1-\frac{x}{R}\right)$

Answers

1. d
2. c
3. a
4. d
5. a
6. b
7. d
8. a, b
9. d
10. a, b
11. a
12. c
13. 5.66 m
14. $10\sqrt{2}$ ms^{-1}
15. 2.49 s
16. 1/3 km
17. 10 ms^{-1}
18. –
19. –
20. –